I. F.S. EXAM-2015

वियोज्य DETACHABLE

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STATISTICS

Paper I

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Question No. **1** and **5** are compulsory. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two **Sections A** and **B**.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the answer book must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in **ENGLISH** only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

SECTION 'A'

1. Answer *all* of the following : 8×5=40

1.(a) Define conditional probability of event A given B. Show that for any three events A, B and C with $P(C) > 0$,

$$P(A \cup B | C) = P(A|C) + P(B|C) - P(A \cap B | C).$$

1.(b) If the random variable X has the probability density function

$$f_X(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & x > 0, \beta > 0; \\ 0, & \text{otherwise}, \end{cases}$$

obtain the distribution of the random variable $\gamma = \left(\frac{2X}{\beta}\right)^{1/2}$.

1.(c) Define the moment generating function (mgf) of a random variable. Obtain the mgf of $X = \sum_{i=1}^{n} X_i$ where X_i follows Poisson distribution with parameter λ_i $(i = 1, 2, ..., n)$. Assume X_i's are independent.

1.(d) Prove : The minimum variance unbiased estimator of a parameter is unique.

1.(e) Define an unbiased test. Show that a most powerful test of a simple hypothesis against a simple alternative is necessarily unbiased.

2.(a) An ecologist wishes to mark off a circular sampling region having radius 10 mts. However, due to reasons of topography etc, radius of actual region marked is a random variable R with probability density function

$$f_R(r) = \begin{cases} \frac{3}{4}\left[1-(10-r)^2\right], & 9 \leq r \leq 11 \\ 0, & \text{otherwise.} \end{cases}$$

What is the expected area of the resulting circular region ? 10

2.(b) Consider the random variable X with probability density function (pdf)

$$f_X(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \quad x > \alpha, \ \alpha > 0, \ \beta > 0.$$

Verify that $f_X(x)$ is a pdf. Show that the distribution is positively skewed. 10

2.(c) Examine whether the WLLN holds for independent random variables with distribution

$$P\left[X_n = \mp\sqrt{n}\right] = \frac{1}{2n}, \ P\left[X_n = 0\right] = 1 - \frac{1}{n}, \ n = 1, 2, \ldots..$$ 10

2.(d) Define characteristic function of a random variable. Show that for two independent random variables X and Y the characteristic functions $\psi_X(t)$ and $\psi_Y(t)$ satisfy the relation $\psi_X(t) \times \psi_Y(t) = \psi_Z(t)$

where $\psi_Z(t)$ is the characteristic function of $Z = X + Y$. 10

3.(a) Let $X_1, X_2, \ldots, X_n$ be a random sample from an exponential distribution with pdf

$$f_X(x; \theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & x > 0, \ \theta > 0; \\ 0, & \text{otherwise.} \end{cases}$$

Find MVB estimator of θ. 10

3.(b) Explaining SPRT, develop it for testing $H_0 : \theta = \frac{1}{3}$ against $H_1 : \theta = \frac{2}{3}$ for the distribution

$$f_X(x, \theta) = \begin{cases} \theta^x (1-\theta)^{1-x}, & x = 0, 1, \quad 0 < \theta < 1; \\ 0, & \text{otherwise.} \end{cases}$$ 10

3.(c) Given a random sample $X_1, X_2, \ldots, X_n$ from a distribution with probability density function

$f_X(x, \theta) = \theta e^{-\theta x}, x > 0, \theta > 0.$

Show that there exists no UMP test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. 10

3.(d) Define a maximum likelihood estimator (MLE) of a parameter. Obtain MLE of σ^2 based on a random sample of size n from a $N(\mu, \sigma^2)$ population when (i) μ is known, and (ii) μ is unknown. Examine whether estimators obtained are unbiased. 10

4.(a) Show that under squared error loss (SEL) function posterior mean is the Bayes' estimator of a parameter.

Let $X \sim b(n, p)$. Assuming n known, obtain the Bayes' estimator of p under SEL when a priori distribution of p is $\pi(p) = 1,\ 0 < p < 1$. 10

4.(b) State and prove the Rao-Blackwell theorem. Explain its usefulness in finding a UMVU estimator of an unknown parameter. 10

4.(c) Show that for Cauchy distribution with location parameter μ and scale parameter σ, having pdf

$$f_X(x) = \frac{1}{\pi\sigma\left(1+\left(\frac{x-\mu}{\sigma}\right)^2\right)} \quad -\infty < x < \infty$$

the three quartiles of the distribution are given by

$Q_1 = \mu - \sigma,\ Q_2 = \mu,\ Q_3 = \mu + \sigma.$ 10

4.(d) Define convergence in probability and convergence in distribution. Show that the former implies the latter. When is the converse true ? Substantiate your answer. 10

SECTION 'B'

5. Answer *all* of the following : 8×5=40

5.(a) Suppose that Y_1, Y_2 and Y_3 are independent random observations with

$E(Y_1) = \theta_1 + \theta_2$, $E(Y_2) = \theta_2 + \theta_3$, $E(Y_3) = \theta_3 - \theta_1$ and $V(Y_i) = \sigma^2$, $i = 1, 2, 3.$

(i) Examine whether θ_2 is estimable.

(ii) Obtain the best estimator for $\theta_1 + 3\theta_2 + 2\theta_3$.

5.(b) The allele of a pea section can be *AA*, *Aa* and *aa* with probabilities p_1, p_2 and p_3 respectively. Let X_1, X_2 and X_3 denote respectively the frequencies of these three categories in a sample of size 10. Write down the joint probability mass function of (X_1, X_2, X_3) and calculate the correlation coefficient between X_1 and X_2.

5.(c) Let $\underline{X}$ be $N_3(\underline{\mu}, \Sigma)$ with $\Sigma = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Which pairs of variables are independent ? Find correlation coefficient between X_1 and X_2.

5.(d) From a bivariate normal $N_2(\underline{\mu}, \Sigma)$ distribution, a random sample of size 3 is drawn. The unbiased estimates of $\underline{\mu}$ and Σ are $\overline{X} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$, $S = \begin{bmatrix} 4 & -3 \\ -3 & 9 \end{bmatrix}$. Compute Hotelling's T^2 and test the null hypothesis $H_0 : \underline{\mu} = \underline{\mu}_0 = \begin{pmatrix} 7 \\ 7 \end{pmatrix}$.

5.(e) Consider two blocks with two plots each and yields as below :

Block 1	y_{11}	y_{12}
Block 2	y_{23}	y_{24}

Show that the treatment contrasts $\tau_1 - \tau_3$, $\tau_1 - \tau_4$, $\tau_2 - \tau_3$ and $\tau_2 - \tau_4$ are not estimable.

6.(a) For a linear model, obtain the least squares estimate of the vector of parameters and show that it is unbiased. 10

6.(b) Define multiple correlation coefficient $R_{1\cdot23\ldots p}$ between x_1 and $x_2, x_3, \ldots, x_p$. Show that $1 - R^2_{1\cdot23\ldots p} = \frac{R}{R_{11}}$ where R_{11} is the minor of the element in the first row and first column of the correlation matrix $R = ((rij))^{pXp}$. 10

6.(c) Define a p-variate normal distribution. When is it said to be singular ? If $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$, $\underline{\mu} = (2, -3, 1)'$ and $\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$, find the distribution of the vector $(X_1 - X_2, X_2 - X_3)'$. 10

6.(d) Explain the problem of discriminant analysis. Describe the allocation rule under Fisher discriminant function. Show that it minimises the misclassification probability. 10

7.(a) What is cluster sampling ? Considering clusters of unequal sizes, suggest an unbiased estimator of the population mean and obtain its variance. 10

7.(b) Consider all possible samples of size 2 from a population of size 3, namely $s_1 = \{1, 2\}$, $s_2 = \{1, 3\}$ and $s_3 = \{2, 3\}$ with probabilities $P(s_i) = \frac{1}{3}$, $i = 1, 2, 3$. Define the estimator

$$t = \begin{cases} \frac{y_1}{2} + \frac{3}{4} y_2 \text{ if } s_1 \text{ is selected,} \\ \\ \frac{1}{2} y_1 + \frac{1}{2} y_3 \text{ if } s_2 \text{ is selected,} \\ \\ \frac{1}{4} y_2 + \frac{1}{2} y_3 \text{ if } s_3 \text{ is selected.} \end{cases}$$

Examine whether t is unbiased. Find its variance. 10

7.(c) Construct an LSD of size 5. Delete one column from the LSD. Prove that the resulting design is a symmetrical BIBD with parameters $v = b = 5$, $r = k = 4$ and $\lambda = 3$. 10

7.(d) Why is a split-plot design needed ? Give two examples to motivate the use of such a design. Present the relevant ANOVA table. 10

8.(a) Why were randomised response techniques devised ? Describe Warner's technique for estimating the population proportion relating to the sensitive character. 10

8.(b) Define an orthogonal contrast. Explain Tukey's test for testing its significance. 10

8.(c) Explain the need for factorial experiments. Develop a method to estimate all main effects of a 2^3 factorial experiment. Give its ANOVA table. 10

8.(d) Define Mahalanobis D^2 statistic. Show that it is invariant. Give an application of this statistic. 10
