C–HENT–N–YPNMA

PHYSICS

Paper—I

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions :

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Question nos. **1** and **5** are compulsory. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.

All questions carry equal marks. The number of marks carried by a part of a question is indicated against it.

Answers must be written in **ENGLISH** only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

All parts and sub-parts of a question are to be attempted together in the answer book.

Any page or portion of the page left blank in the Answer Book must be clearly struck out.

Useful Constants

Electron charge (e)	=	1.602×10^{-19} C
Electron rest mass (m_e)	=	9.109×10^{-31} kg
Proton mass (m_p)	=	1.672×10^{-27} kg
Vacuum permittivity (ε_0)	=	8.854×10^{-12} farad/m
Vacuum permeability (μ_0)	=	1.257×10^{-6} henry/m
Velocity of light in free space (c)	=	3×10^{8} m/s
Boltzmann constant (k)	=	1.38×10^{-23} J/K
Electron volt (eV)	=	1.602×10^{-19} J
Planck's constant (h)	=	6.62×10^{-34} J-s
Stefan's constant (σ)	=	5.67×10^{-8} Wm^{-2} K^{-4}
Avogadro's number (N)	=	6.02×10^{26} kmol^{-1}
Gas constant (R)	=	8.31×10^{3} J kmol^{-1} K^{-1}
exp (1)	=	2.7183

SECTION—A

Q. 1. Answer the following : **8×5=40**

Q. 1(a) Using the concept of D'Alembert's principle, show that the generalized force can be defined as

$$Q_j = \sum_i \vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j},$$

where $\vec{r}_i$ is the Cartesian coordinate of the i^{th} particle experiencing external force $\vec{F}_i$ and q_j stands for the generalized coordinate. Discuss the significance of the above expression. 8

Q. 1(b) Write down the mathematical representation of Fermat's principle and explain all the notations used.

With the help of a neat diagram, show that this principle can be used to obtain the law of refraction :

$$n_1 \sin \theta_1 = n_2 \sin \theta_2,$$

where n_1 and n_2 are the refractive indices of the two media while θ_1 and θ_2 are the angles of incidence and refraction of the light beam. 8

Q. 1(c) If the system matrix for a thin lens of focal length f is given by :

$$S = \begin{bmatrix} 1 & -1/f \\ 0 & 1 \end{bmatrix},$$

show that the system matrix for a combination of two thin lenses of focal lengths f_1 and f_2 separated by a distance d can be obtained as

$$S_{12} = \begin{bmatrix} 1-d/f_2 & -(1/f_1 + 1/f_2 - d/f_1 f_2) \\ d & 1-d/f_1 \end{bmatrix}.$$ 8

Q. 1(d) A left circularly polarized beam (λ = 5893°A) is incident normally on a calcite crystal (with its optic axis cut parallel to the surface) of thickness 0.005141 mm. What will be the state of polarization of the emergent beam ? Justify your answer. Here the refractive indices for the ordinary and the extraordinary rays are 1.65836 and 1.48641 respectively. 8

Q. 1(e) Deduce the minimum energy of a gamma ray photon (in MeV), which can cause electron-positron pair production. 8

Q. 2(a) State Hamilton's principle for a system of particles. If I and L represent the action integral and the Lagrangian function, respectively, write down the mathematical form of Hamilton's principle and explain clearly the significance of the same. 15

Q. 2(b) Discuss the torque-free motion of a symmetric rigid body and hence estimate the period of precession of Earth's polar axis. [Use $\frac{I_3 - I_1}{I_3} = 0.33$, where I_1, I_2 and I_3 are the moments of inertia about the three principal axes of the Earth] 15

Q. 2(c) A particle of rest mass M moving at a velocity u collides with a stationary particle of rest mass m. If the particles stick together, show that the speed of the composite ball is equal to $u\alpha M/(\alpha M + m)$, where $\alpha = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$. 10

Q. 3(a) Derive the law of addition of relativistic velocities. Use it to prove that under the Lorentz transformation no two velocities can add upto more than the value of the speed of light. 15

Q. 3(b) Starting with the rate equations for matter-radiation interaction, show that the ratio of the Einstein's A and B coefficients is proportional to the third power of the frequency of radiation. 15

Q. 3(c) How does the population inversion in an active medium lead to the amplification of light in a laser ? Explain in details. 10

Q. 4(a) Explain how you can construct a zone plate from Fresnel's half period zone. Show that a zone plate has multiple foci. 15

Q. 4(b) What are the step index and graded index optical fibers ? What are the conventional optical windows for light propagation through optical fibers ? 15

Q. 4(c) Draw a neat diagram to show how light propagates through a graded index optical fiber from the entrance to the fiber end and explain. 10

SECTION—B

Q. 5. **Answer the following :** **8×5=40**

Q. 5(a) Prove Stefan's law of radiation from thermodynamic considerations. 8

Q. 5(b) For a degenerate Fermi-Dirac gas, the concentration of nucleons in nuclear matter is $N = 1.1 \times 10^{38}$ cm^{-3}. Calculate the Fermi energy and Fermi temperature. 8

Q. 5(c) Suppose a cavity of volume V contains blackbody radiation in equilibrium with the walls of the cavity at a temperature T. For a reversible adiabatic change of volume show that :

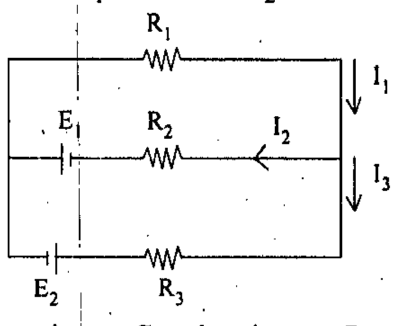
VT^3 = constant.

If the initial temperature is 2000°K and the volume is increased from 10 cm^3 to 1250 cm^3, reversibly and adiabatically, what would be the final temperature of the radiation ? 8

Q. 5(d) Compute the electrostatic energy of a conducting solid sphere of radius R and having total charge Q using the expression for the electrostatic energy in terms of the electric field $\vec{E}$ of the above sphere. 8

Q. 5(e) Show that the elemental quantity of heat dQ is not a total differential. 8

Q. 6(a) Explain Kirchoff's circuit laws. Using Kirchoff's laws find currents I_1, I_2 and I_3 in the circuit shown below for $R_1 = 100\ \Omega$, $R_2 = 200\ \Omega$, $R_3 = 300\ \Omega$, $E_1 = 3$ V, $E_2 = 4$ V. 10



Q. 6(b) An inductance L, capacitance C and resistance R are connected in series to form a circuit with AC source driving with $E_{rms} = 120$ V at f = 60 Hz. Compute the power factor and average power dissipated in the resistance if $R = 200\ \Omega$, $X_L = 80\ \Omega$ and $X_C = 150\ \Omega$. 15

Q. 6(c) A sphere of homogeneous linear dielectric material is placed in an otherwise uniform electric field $\vec{E}_0$. Find the electric field inside the sphere. 15

Q. 7(a) A plane electromagnetic wave of frequency w_1 travelling in z-direction and polarized in x-direction is incident on a dielectric medium separated by a plane boundary in the x-z plane. Show that the transmission and reflection coefficient are given by :

$$R = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}, \quad T = \frac{4n_1 n_2}{(n_1 + n_2)^2},$$

where n_1 and n_2 are the refractive indices of the first and the second medium. 20

Q. 7(b) (i) Write Maxwell's equations in the absence of a medium.

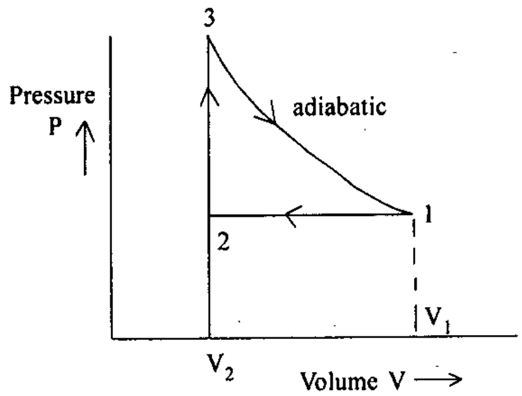
(ii) Explain the gauge transformation and gauge invariance. Show that the relation $\text{div}\ \vec{A} + \frac{1}{c}\frac{\partial \phi}{\partial t}$ is Lorentz-covariant, where $\vec{A}$ and ϕ are the vector potential and the scalar potential respectively.

(iii) Obtain Maxwell's wave equations satisfied by the scalar and vector potentials in the absence of charge and current. 5+5+10

Q. 8(a) Derive an expression for the Fermi energy for a free electron gas at T = 0. Compute the Fermi temperature of Cu assuming the density 9 gms/cm^3 and one conduction electron per atom. 5

Q. 8(b) The figure below represents an imaginary ideal gas cycle. Assuming constant heat capacities, show that the thermal efficiency is :

$$\eta = 1 - Y\frac{(V_1/V_2)-1}{(P_3/P_2)-1}.$$ 15



Q. 8(c) A parallel plate capacitor is connected to a battery. What is the electric current while the capacitor is being charged ? What is the displacement current between the plates of the capacitor ? Show that the rate of increase of the electric energy is equal to the surface integral of the Poynting vector over the surface enclosing the volume between the plates of the capacitor. 20