I.F.S. EXAM-2016

वियोज्य *DETACHABLE*

MATHEMATICS

Paper II

0000129

Time Allowed : Three Hours	Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Question Nos. **1** and **5** are compulsory. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two **Sections A** and **B**.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in **ENGLISH** only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

SECTION 'A'

1.(a) Prove that the set of all bijective functions from a non-empty set X onto itself is a group with respect to usual composition of functions. 8

1.(b) Examine the Uniform Convergence of

$$f_n(x) = \frac{\sin(nx+n)}{n}, \forall x \in \mathbb{R},\ n = 1, 2, 3, \ldots$$ 8

1.(c) Find the maxima and minima of the function
$f(x, y) = x^3 + y^3 - 3x - 12y + 20.$ 8

1.(d) Find the analytic function of which the real part is

$$e^{-x}\left\{\left(x^2 - y^2\right)\cos y + 2xy\sin y\right\}.$$ 8

1.(e) Prove that the set of all feasible solutions of a Linear Programming problem is a convex set. 8

2.(a) Show that any non-abelian group of order 6 is isomorphic to the symmetric group S_3. 15

2.(b) Let G be a group of order pq, where p and q are prime numbers such that $p > q$ and $q \nmid (p-1)$. Then prove that G is cyclic. 15

2.(c) Show that in the ring $R = \left\{a + b\sqrt{-5} \mid a, b \text{ are integers}\right\}$, the elements $\alpha = 3$ and $\beta = 1 + 2\sqrt{-5}$ are relatively prime, but $\alpha\gamma$ and $\beta\gamma$ have no g.c.d in R, where $\gamma = 7\left(1 + 2\sqrt{-5}\right)$. 10

3.(a) If $f_n(x) = \frac{3}{x+n}$, $0 \le x \le 2$, state with reasons whether $\{f_n\}_n$ converges uniformly on [0, 2] or not. 10

3.(b) Examine the continuity of $f(x, y) = \begin{cases} \dfrac{\sin^{-1}(x+2y)}{\tan^{-1}(2x+4y)}, & (x, y) \neq (0, 0) \\ \frac{1}{2}, & (x, y) = (0, 0) \end{cases}$

at the point (0, 0). 8

3.(c) If $u(x, y) = \cos^{-1}\left\{\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right\}$, $0 < x < 1$, $0 < y < 1$ then find the value of

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}.$$ 10

3.(d) Evaluate the integral $\displaystyle\int_0^2\int_0^{y^2/2} \frac{y}{\left(x^2 + y^2 + 1\right)^{\frac{1}{2}}}\, dx\, dy.$ 12

4.(a) Evaluate the integral $\int_{0}^{\infty} \frac{dx}{\sqrt{x}(1+x)}$. 8

4.(b) Find the Laurent series for the function $f(z)=\frac{1}{1-z^2}$ with centre $z=1$. 10

4.(c) Evaluate by Contour integration $\int_{0}^{\pi} \frac{d\theta}{\left(1+\frac{1}{2}\cos\theta\right)^2}$. 10

4.(d) A company manufacturing air-coolers has two plants located at Bengaluru and Mumbai with a weekly capacity of 200 units and 100 units respectively. The company supplies air-coolers to its 4 showrooms situated at Mangalore, Bengaluru, Delhi and Goa which have a demand of 75, 100, 100 and 25 units respectively. Due to the differences in local taxes, showroom charges, transportation cost and others, the profits differ. The profits (in Rs.) are shown in the following table :

From	To			
	Mangalore	Bengaluru	Delhi	Goa
Bengaluru	90	90	100	100
Mumbai	50	70	130	85

Plan the production program so as to maximize the profit. The company may have its production capacity at both plants partially or wholly unused. 12

SECTION 'B'

5.(a) Obtain the partial differential equation governing the equations

$$\phi(u, v) = 0, \quad u = xyz,$$
$$v = x + y + z.$$ 8

5.(b) Find the general solution of the partial differential equation

$$xy^2 \frac{\partial z}{\partial x} + y^3 \frac{\partial z}{\partial y} = \left(zxy^2 - 4x^3\right).$$ 8

5.(c) Develop an algorithm for Newton-Raphson method to solve $\phi(x) = 0$ starting with initial iterate x_0, n be the number of iterations allowed, eps be the prescribed relative error and delta be the prescribed lower bound for $\phi'(x)$. 8

5.(d) Apply Lagrange's interpolation formula to find $f(5)$ and $f(6)$ given that $f(1)=2$, $f(2)=4$, $f(3)=8$, $f(7)=128$. 8

5.(e) Calculate the moment of inertia of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(i) relative to the x-axis
(ii) relative to the y-axis and
(iii) relative to the origin 8

6.(a) Find the general solution of the partial differential equation

$$xy^2p + y^3q = (zxy^2 - 4x^3)$$

$$\left[p=\frac{\partial z}{\partial x},\ q=\frac{\partial z}{\partial y}\right]$$ 10

6.(b) Find the particular integral of $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x \partial y}+\frac{\partial^2 z}{\partial y^2}=2x\cos y$. 10

6.(c) A uniform rod of length L whose surface is thermally insulated is initially at temperature $\theta = \theta_0$. At time $t=0$, one end is suddenly cooled to $\theta = 0$ and subsequently maintained at this temperature; the other end remains thermally insulated. Find the temperature distribution $\theta(x, t)$. 20

7.(a) Evaluate $\int_0^{0\cdot 6} \frac{dx}{\sqrt{1-x^2}}$ by Simpson's $\frac{1}{3}$rd rule, by taking 12 equal sub-intervals. 15

7.(b) Find the cube root of 10 up to 5 significant figures by Newton-Raphson method. 10

7.(c) Use the Classical Fourth-order Runge-Kutta method with $h=\cdot 2$ to calculate a solution at $x=\cdot 4$ for the initial value problem $\frac{dy}{dx}=x+y^2$ with initial condition $y=1$ when $x=0$. 15

8.(a) Find the moment of inertia of a right solid cone of mass M, height h and radius of whose base is a, about its axis. 14

8.(b) A bead slides on a wire in the shape of a cycloid described by the equations

$$x = a(\theta - \sin\theta)$$
$$y = a(1 + \cos\theta)$$

where $0 \leq \theta \leq 2\pi$ and the friction between the bead and the wire is negligible. Deduce Lagrange's equation of motion. 10

8.(c) A sphere is at rest in an infinite mass of homogeneous liquid of density ρ, the pressure at infinity being P. If the radius R of the sphere varies in such a way that $R = a + b\cos nt$, where $b < a$, then find the pressure at the surface of the sphere at any time. 16